Analysis of Cylindrical Stripline with Multilayer Dielectrics

C. JAGADESWARA REDDY AND MONOHAR D. DESHPANDE

Abstract —An inhomogeneous, multidielectric-layered cylindrical stripline using TEM-model approximation is analyzed. A simple, closed-form expression for the characteristic impedance of an inhomogenous cylindrical stripline is derived. Numerical results on the characteristic impedance are compared with the results obtained by Wang [1], and also with the results obtained by the conformal transformation [2]. Extensive data on the impedances of inhomogeneous cylindrical stripline are also presented.

I. Introduction

A CYLINDRICAL STRIPLINE consisting of a circular arc strip placed between two cylindrical ground planes separated by a dielectric material has been analyzed by many workers [1]–[3]. Assuming only TEM mode exists, Wang [1] presented extensive results on the characteristic impedance of such lines. By setting up dual series equations, the constants appearing in the solution of the Laplace equation were evaluated using the Least Square Method or Simple Integration Method. However, the results in [1] are valid for homogeneous dielectric medium. Further, the dual series equations are to be solved each time the transmission line structure is changed. Applying conformal transformation, Rao *et al.* [2] have analyzed cylindrical striplines with homogeneous medium. By using the logarithmic transform, a cylindrical strip was transformed to a planar strip with a finite ground planes. Assuming the strip width to be very small (compared to 2π) the characteristic impedance of the resulting planar strip was found using the well-known conformal transformation. Hence, the method [2] is expected to give correct results for very small strip widths. Solving the Laplace equation in an elliptical coordinate system, Joshi *et al.* [3] have solved the problem of cylindrical stripline for a homogeneous dielectric medium. In this work, a method of analyzing cylindrical stripline on a multilayer dielectric-coated cylinders is presented.

In this paper, assuming TEM-mode field, the Laplace equation is solved for potential distribution function $\Psi(\rho,\Phi)$ in the various dielectric regions. The constants appearing in the solutions are evaluated by subjecting $\Psi(\rho,\Phi)$ to the proper boundary conditions. From the $\Psi(\rho,\Phi)$ function, the electromagnetic field and hence the properties of cylindrical stripline are determined. A special feature of the present method is that by using a reasonable approximation, an expression for the characteristic impedance of a multilayer cylindrical stripline is obtained in a closed form. The results obtained by the present method are compared with the available results [1], [2].

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C. Jagadeswara Reddy is with the Center for Radar and Communication, India Institute of Technology, Kharagpur- 721302, India.
M. D. Deshpande is with the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur-721302, India.
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II. Theory

The cross section of a cylindrical stripline to be analyzed is shown in Fig. 1, with the notation to be used. Assuming only TEM-mode field, the potential function $\Psi(\rho,\Phi)$ in various regions satisfies the Laplace equation [4]

$$\rho\frac{\delta}{\delta\rho}\left(\rho\frac{\delta\Psi}{\delta\rho}\right)+\frac{\delta^2\Psi}{\delta\Phi^2}=0. \tag{1}$$

Using the method of variable separation, the solution of (1) can be written as [4]

$$\Psi_k(\rho,\Phi)=A_{k0}\ln\rho+B_{k0}+\sum_n\left(A_{kn}\rho^n+B_{kn}\rho^{-n}\right)\cos n\Phi \tag{2}$$

where the subscript k becomes 1 for the region I (i.e., $a\leqslant\rho\leqslant d$), 2 for the region II (i.e., $d\leqslant\rho\leqslant b$), and 3 for the region III (i.e., $b\leqslant\rho\leqslant c$). Subjecting the $\Psi_k(\rho,\Phi)$ given in (2) to the following boundary conditions:

$$\Psi_1(a,\Phi)=0 \tag{3}$$

$$\Psi_3(c,\Phi)=0 \tag{4}$$

$$\Psi_1(d,\Phi)=\Psi_2(d,\Phi) \tag{5}$$

$$\frac{\delta\Psi_1(d,\Phi)}{\delta\Phi}=\frac{\delta\Psi_2(d,\Phi)}{\delta\Phi} \tag{6}$$

$$\Psi_2(b,\Phi)=\Psi_3(b,\Phi) \tag{7}$$

the potential function Ψ_k for three regions takes the form

$$\Psi_1(\rho,\Phi)=a_0\ln(\rho/a)+\sum_n a_n\sinh(n\ln\rho/a)\cos n\Phi,\quad \text{for } a\leqslant\rho\leqslant d \tag{8}$$

$$\Psi_2(\rho,\Phi)=b_0\ln(\rho/d)-b_0'\ln(\rho/b)+\sum_n a_n\sinh(n\ln\rho/b)-b_n\sinh(n\ln\rho/d)\cos n\Phi,\quad \text{for } d\leqslant\rho\leqslant b \tag{9}$$

$$\Psi_3(\rho,\Phi)=c_0\ln(\rho/c)+\sum_n c_n\sinh(n\ln\rho/c)\cos n\Phi,\quad \text{for } b\leqslant\rho\leqslant c. \tag{10}$$

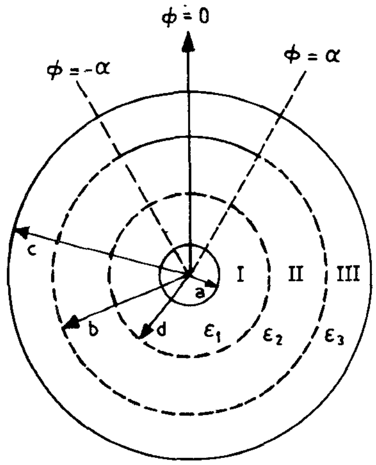

Fig. 1. Cross section of cylindrical stripline.

In the above equations

$$a_0 = A_{10}, \qquad a_n = 2A_{1n}a^n \tag{11}$$

$$b_0 = a_0\left[\sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}} + \frac{\ln(d/a)}{\ln(b/d)}\right] \tag{12}$$

$$b_0' = a_0\frac{\ln(d/a)}{\ln(b/d)} \tag{13}$$

$$b_n = \sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}} + M_n\frac{\cosh(n\ln d/b)}{\cosh(n\ln d/a)} \tag{14}$$

$$c_0 = \frac{a_0}{\ln(b/c)}\left[\sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}}\ln b/d + \ln d/a\right] \tag{15}$$

$$c_n = -\frac{a_n}{2N_n}\left[\sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}} + M_n\frac{\cosh(n\ln d/b)}{\cosh(n\ln d/a)}\right] \tag{16}$$

$$M_n = \frac{\sinh(n\ln d/a)}{\sinh(n\ln b/a)}$$

$$N_n = \frac{\sinh(n\ln c/b)}{\sinh(n\ln b/d)}. \tag{17}$$

It may be noted here that the present method up to this point is an extension of Wang's method [1] to a multilayer dielectric-coated cylinder. The constants a_o and a_n may be evaluated by setting a dual series as is done in [1] and then using the Least Square Method or Simple Integration Method. However in the present work, the method described below to evaluate a_o and a_n is followed. For small values of $(c-a)$, the electromagnetic field would be confined to the region just below the strip and inner cylinder (i.e., $\rho = a$). Taking the fringing field into account, the potential at $\rho = b$ is therefore

$$V(\Phi) = \begin{cases} 1, & -\alpha-\Delta\alpha < \Phi < \alpha+\Delta\alpha \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

where $\Delta\alpha$ is the extra half-strip angle to account for the fringing field.

The unknown coefficients a_o and a_n, using the condition

$$\Psi_2(b,\Phi) = \Psi_3(b,\Phi) = V(\Phi) \tag{19}$$

are obtained as

$$a_o = \frac{\alpha}{\pi\left(\sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}}\ln b/d + \ln d/a\right)} \tag{20}$$

$$a_n = \frac{2\sin n\alpha\cosh(n\ln d/a)}{\left(\sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}}\cosh(n\ln d/a) + M_n\cosh(n\ln d/b)\right)}. \tag{21}$$

Following the method described in [1], the characteristic impedance of the cylindrical strip is obtained as

$$\sqrt{\epsilon_{r_1}}\,z_0 = \frac{188.5}{\alpha}\Bigg/\left[\frac{1}{\left(\sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}}\ln b/d + \ln d/a\right)} + \frac{\sqrt{\frac{\epsilon_{r_3}}{\epsilon_{r_1}}}}{(\ln c/b)}\right]. \tag{22}$$

A. Comparison with Wang's method

In order to compare the expression in (22) with the one reported by Wang, the few steps given in [1] are reproduced below. For a cylindrical strip with homogeneous medium and having relative dielectric constant ϵ_r, the unknown coefficients satisfy the dual series

$$a_o g + \sum_n a_n L_n \cos n\Phi = 1, \qquad 0 < \Phi < \alpha \tag{23}$$

$$a_o f + \sum_n a_n K_n \cos n\Phi = 0, \qquad \alpha < \Phi < \pi \tag{24}$$

where g, f, L_n, K_n are as defined in [1, eqs. (6) and (7)]. Following the simple integration method, the unknown a_o is obtained as

$$a_o = \alpha p/g - p\sum_n a_n\left(\frac{L_n}{g} - K_n\right)\frac{\sin n\alpha}{n} \tag{25}$$

where

$$p = [\alpha + (\pi-\alpha)f]^{-1}.$$

For small values of $(c-a)$, $L_n = n\ln(b/a)$, $K_n \cong n$, $f = 1$ and hence

$$a_o = \frac{\alpha}{\pi\ln b/a}. \tag{26}$$

The characteristic impedance of the homogeneous cylindrical strip is then obtained using (8), (9) or [1] as

$$\sqrt{\epsilon_r}\,z_0 = \frac{188.5}{\alpha}\,\frac{\ln(c/b)\ln(b/a)}{\ln(c/a)}. \tag{27}$$

It is to be observed that for $\epsilon_{r_1} = \epsilon_{r_2} = \epsilon_{r_3} = \epsilon_r$, the expression in (22) reduces to the form given in (27).

B. Warped Stripline

In order to study the effect of warpage on an otherwise planar structure with multilayer dielectrics, we consider a structure as shown in Fig. 2 where $c-b = b-a = h/2$, $b-d = d-a = h/4$, $c-a = h$. The strip width $W = 2\alpha b$,

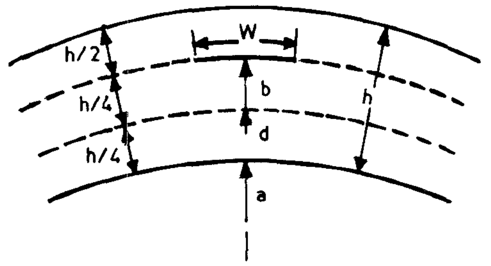

Fig. 2. Cross section of warped stripline.

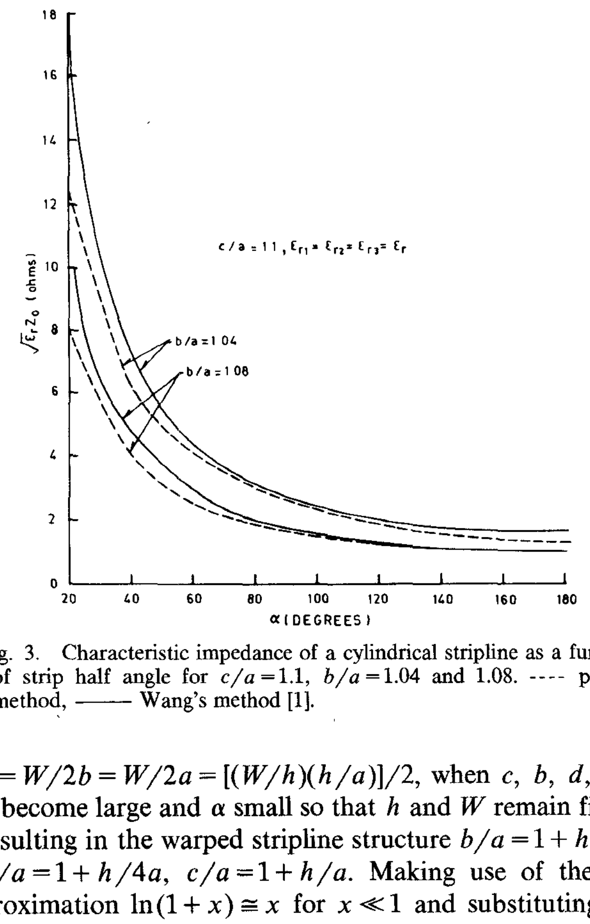

Fig. 3. Characteristic impedance of a cylindrical stripline as a function of strip half angle for $c/a=1.1$, $b/a=1.04$ and 1.08. ---- present method, —— Wang's method [1].

$\alpha = W/2b = W/2a = [(W/h)(h/a)]/2$, when c, b, d, and a become large and α small so that h and W remain finite, resulting in the warped stripline structure $b/a=1+h/2a$, $d/a=1+h/4a$, $c/a=1+h/a$. Making use of the approximation $\ln(1+x) \cong x$ for $x \ll 1$ and substituting the above relations in (22), the impedance formula can be obtained as

$$\sqrt{\epsilon_{r_1}}\, z_o = \frac{94.25}{(w/h)} \Bigg/ \left[\frac{1}{1+\sqrt{\dfrac{\epsilon_{r_1}}{\epsilon_{r_2}}}} + \frac{1}{2}\sqrt{\frac{\epsilon_{r_3}}{\epsilon_{r_1}}} \right]. \quad (28)$$

For a homogeneous case considered by Wang [1] $\epsilon_{r_1} = \epsilon_{r_2} = \epsilon_{r_3} = \epsilon_r$, the impedance formula reduces to a simple form (when $h/a \ll 1$)

$$\sqrt{\epsilon_r}\, z_0 = \frac{94.25}{(w/h)} \quad (29)$$

where warpage is indicated by h/a.

III. Numerical Results

To verify the validity of the present formulation, the characteristic impedance of a cylindrical strip with $\epsilon_{r_1} = \epsilon_{r_2} = \epsilon_{r_3} = \epsilon_r$, $c/a=1.1$, $b/a=1.04$ and 1.08 is computed

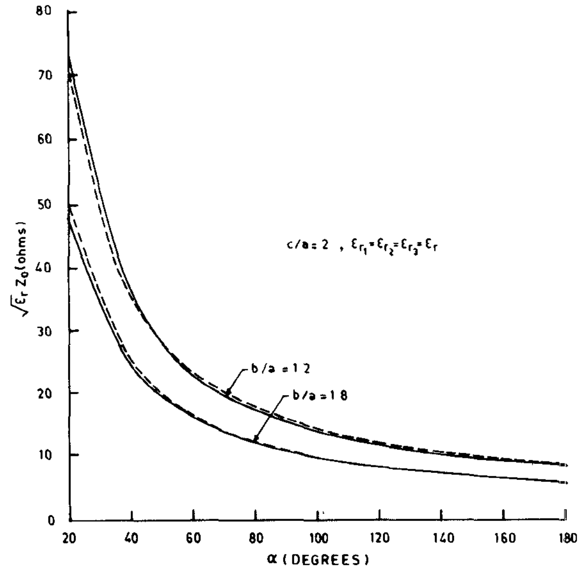

Fig. 4. Characteristic impedance of cylindrical stripline as a function of strip half angle for $c/a=2$, $b/a=1.2$ and 1.8. ---- present method, —— Wang's method [1].

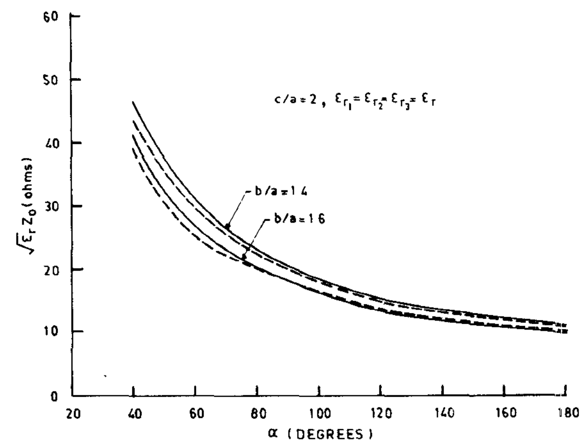

Fig. 5. Characteristic impedance of cylindrical stripline as a function of strip half angle for $c/a=2$, $b/a=1.4$ and 1.6. ---- present method, —— Wang's method [1].

TABLE I
COMPARISION OF a_o VALUES CALCULATED BY THE PRESENT METHOD AND WANG'S METHOD [1]

α (Degrees)	c/a = 1.1 b/a=1.04		c/a = 2 b/a=1.8		c/a = 4 b/a=3.9		c/a=10 b/a=*.2	
	Present Method	Wang	Present Method	Wang	Present Method	Wang	Present Method	Wang
20	2.8330	2.8331	0.1890	0.1886	0.0816	0.0822	0.6094	0.6423
40	5.6659	5.6660	0.3781	0.3793	0.1633	0.1637	1.2188	1.2706
60	8.4989	8.4989	0.5671	0.5672	0.2449	0.2446	1.8283	1.8995
80	11.3319	11.3320	0.7561	0.7583	0.3266	0.3269	2.4377	2.5012
100	14.1649	14.1649	0.9452	0.9477	0.4082	0.4097	3.0471	3.1437
120	16.9978	16.9948	1.1342	1.1403	0.4898	0.4912	3.6565	3.7602
140	19.8308	19.8316	1.3232	1.3330	0.5715	0.5736	4.2660	4.3981
160	22.6638	22.6703	1.5123	1.5300	0.6531	0.6561	4.8754	4.9871
180	25.4968	25.4968	1.7013	1.7013	0.7348	0.7348	5.4848	5.4848

using (22) as a function of strip angle α. The results are presented in Fig. 3 along with the available results [1]. There is a very good agreement between the two results. The characteristic impedance of cylindrical stripline for larger values of c/a and b/a is also computed and presented in Figs. 4 and 5, along with the results obtained by

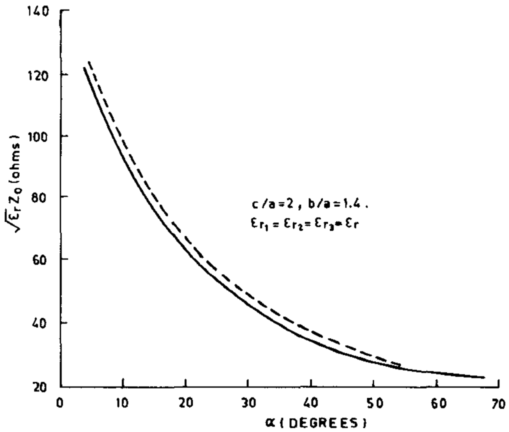

Fig. 6. Characteristic impedance of a cylindrical stripline versus strip half angle with $c/a = 2$, $b/a = 1.4$. —— present method, ---- Joshi *et al.* [3].

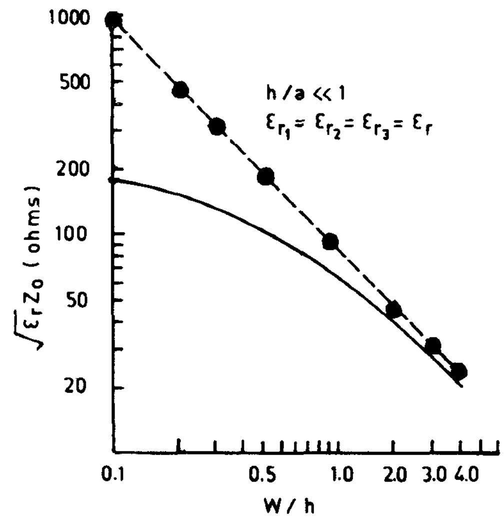

Fig. 7. Characteristic impedance of a warped stripline versus w/h with $h/a \ll 1$. ---- present method, —— planar structure, ··· Wang's method [1].

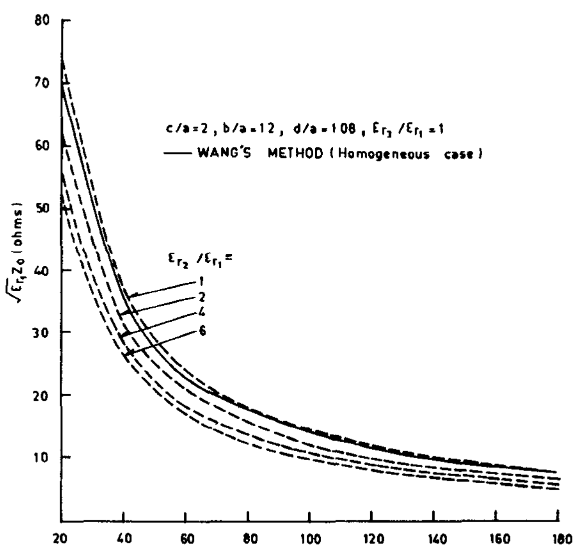

Fig. 8. Characteristic impedance of cylindrical stripline versus strip half angle for $c/a = 2$, $b/a = 1.2$, $d/a = 1.08$, $\epsilon_{r_3}/\epsilon_{r_1} = 1$ and $\epsilon_{r_2}/\epsilon_{r_1} = 1$, 2, 4, and 6.

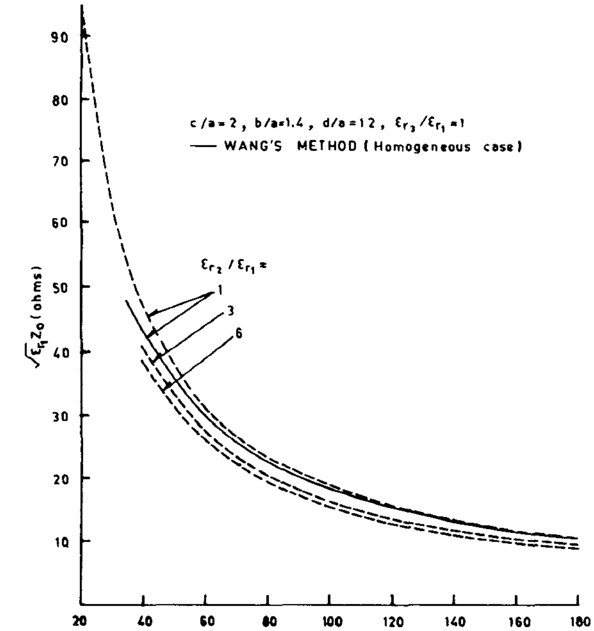

Fig. 9. Characteristic impedance of cylindrical stripline versus strip half angle for $c/a = 2$, $b/a = 1.4$, $d/a = 1.2$, $\epsilon_{r_3}/\epsilon_{r_1} = 1$ and $\epsilon_{r_2}/\epsilon_{r_1} = 1$, 3, and 6.

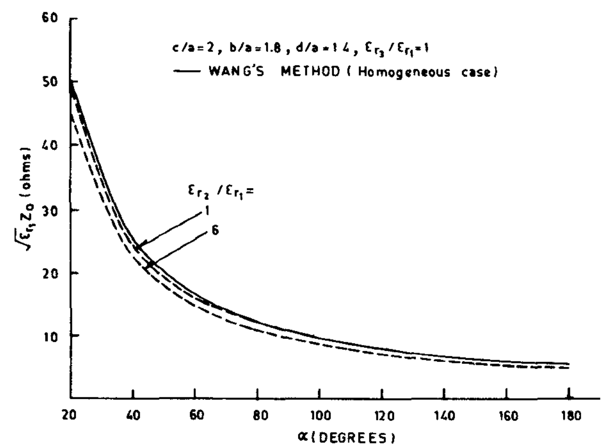

Fig. 10. Characteristic impedance of cylindrical stripline versus strip half angle for $c/a = 2$, $b/a = 1.8$, $d/a = 1.4$, $\epsilon_{r_3}/\epsilon_{r_1}$, and $\epsilon_{r_2}/\epsilon_{r_1} = 1$ and 6.

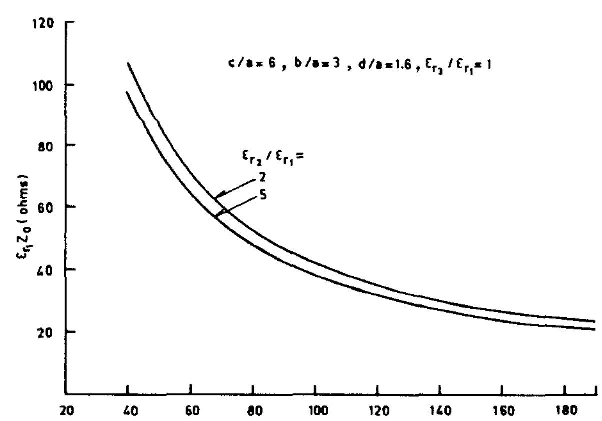

Fig. 11. Characteristic impedance of cylindrical stripline v versus strip half angle with $c/a = 6$, $b/a = 3$, $d/a = 1.6$, $\epsilon_{r_3}/\epsilon_{r_1} = 1$, $\epsilon_{r_2}/\epsilon_{r_1} = 2$ and 5.

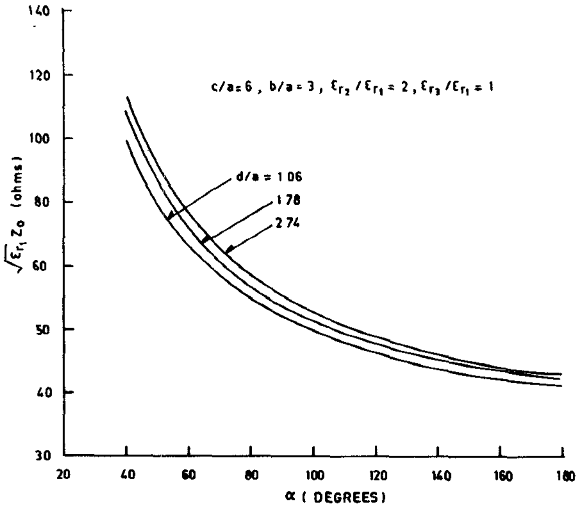

Fig. 12. Characteristic impedance of cylindrical stripline versus strip half angle with $c/a=6$, $b/a=3$, $\epsilon_{r_2}/\epsilon_{r_1}=1$, $d/a=1.06$, 1.78, and 2.74.

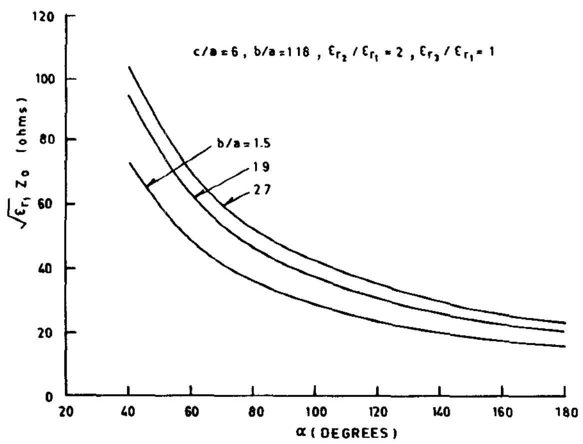

Fig. 13. Characteristic impedance of cylindrical stripline versus strip half angle with $c/a=6$, $d/a=1.18$, $\epsilon_{r_2}/\epsilon_{r_1}=2$, $\epsilon_{r_3}/\epsilon_{r_1}=1$, and $b/a=$ 1.5, 1.9, and 2.7.

Wang [1]. The results obtained by the present method almost coincided with the results obtained by Wang [1]. In order to determine the maximum value of c/a and b/a up to where the present analysis is valid, the coefficients a_o and a_n are calculated by using the simple integration method [1] and the present method. The results of the calculation are shown in Table I. From Table I it is clear that even for $c/a \geq 10$ and $b/a > 2$, error in the present method is around 5 percent.

Taking into account the fringing field by considering the effective width of the strip ($\alpha_{\text{eff}} = \alpha + (b-a)/2b$), the characteristic impedance by the present methods is calculated and compared with the results obtained by Joshi *et al.* [3] in Fig. 6. There is a good agreement between the two results.

Using (29), the characteristic impedance of a warped stripline is computed and presented in Fig. 7 along with the earlier reported results [1]. There is a good agreement between the two results. From Figs. 3–7 it is clear that one

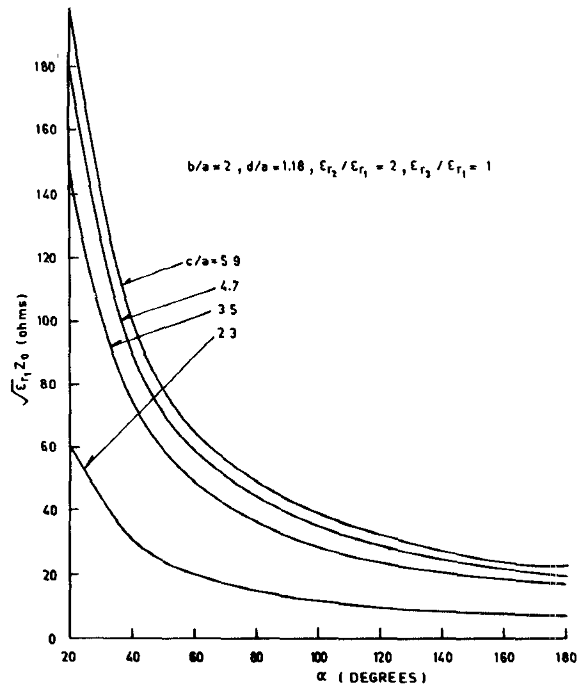

Fig. 14. Characteristic impedance of cylindrical stripline versus strip half angle with $b/a=2$, $d/a=1.18$, $\epsilon_{r_2}/\epsilon_{r_1}=2$, $\epsilon_{r_3}/\epsilon_{r_1}=1$, and c/a = 2.3, 3.5, 4.7, and 5.9.

can use the simple expression given in (22) to find the characteristic impedance of a cylindrical stripline.

Figs. 8–10 show the variation of the characteristic impedance as a function of a half-strip angle for different ratios of $\epsilon_{r_2}/\epsilon_{r_1}$ with $c/a=2$, $b/a=1.2$, 1.4, and 1.8, $d/a=1.08$, 1.2, and 1.4, and $\epsilon_{r_3}/\epsilon_{r_1}=1$. It can be seen from these figures that by using inhomogeneous media, it is possible to vary the impedance for the same half-strip angle. Figs. 11, 13, and 14 show the variation of the impedance with strip half angle for variation of one parameter keeping the others constant. For Figs. 11–14, the particular set of parameters have been chosen to have impedances around 50 Ω.

It can be observed that the characteristic impedance increases as d/a or b/a or c/a is increased as shown in Figs. 12–14, and it decreases as $\epsilon_{r_2}/\epsilon_{r_1}$ increases when other parameters are kept constant. These curves are useful in designing a 50-Ω cylindrical stripline with inhomogeneous media.

IV. Conclusion

A simple, closed-form expression that gives remarkably accurate results for the characteristic impedance of an inhomogeneous cylindrical stripline is obtained. The impedance formula has been simplified for the case of warped stripline assuming large radii and keeping $(c-a)$ small. The numerical results obtained by the present method are compared with the earlier available results and found to be in good agreement. Extensive data on the characteristic impedance of an inhomogeneous cylindrical stripline for various stripline parameters are presented.

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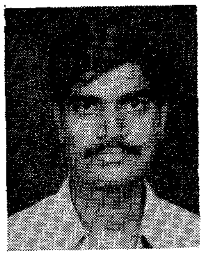
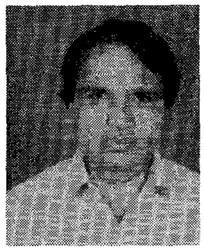


C. Jagadeswara Reddy was born in Nellore, India, on April 17, 1962. He received the B.Tech. degree in electronics and communication engineering from Regional Engineering College, Warangal, India, in 1983; and received the M. Tech. degree in microwave and optical communication engineering from the Indian Institute of Technology, Kharagpur, India, in 1986. Presently, he is working towards the Ph.D. degree at IIT, Kharagpur.



Manohar D. Deshpande was born in Lohgaon, India on June 3, 1948. He received the B.E. (electrical) degree in 1970 from V.R.C.E., Nagpur, and the M.Tech. and Ph.D. degrees in microwave and radar engineering from I.I.T., Kharagpur, India, in 1972 and 1980, respectively.

In 1975, he joined the faculty of the Indian Institute of Technology, Kharagpur, and currently holds the position of Assistant Professor of Electronics & Electrical Communication Engineering.